الجمعية المغربية لعلوم الرياضيات

Moroccan Association of Mathematical Sciences

MAMS

Assignment 1

Instructions	تعليمات
1) This assignment consists of 10 questions: **Section A:** requires numerical answers only. **Section B:** requires full solutions.	1) يتكون هذا الواجب من 10 أسئلة : **القسم A :** يتطلب إجابات عددية فقط. (7 أسئلة) **القسم B :** يتطلب حلولًا كاملة. (3 أسئلة)
2) Each question in Section A is worth 5 points. No partial credit are given, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found.	2) كل سؤال في **القسم A** يساوي **5 نقاط**. لا تُمنح نقاط جزئية. ويجب ألا تعطي أكثر من عدد الإجابات المطلوب. بالنسبة للأسئلة التي تطلب عدة إجابات، تُمنح الدرجة الكاملة فقط إذا تم العثور على جميع الإجابات الصحيحة.
3) Each question in Section B is worth 20 points. Partial credits may be awarded.	3) كل سؤال في **القسم B** يساوي **20 نقطة**. يمكن منح نقاط جزئية.
4) Diagrams shown may not be drawn to scale.	4) قد لا تكون الرسوم التوضيحية المرفقة مرسومة على مقياس صحيح.
5) You cannot use instruments such as protractors, calculators and electronic devices, smart watches.	5) لا يمكنك استخدام أدوات مثل المنقلة، الآلات الحاسبة، الأجهزة الإلكترونية أو الساعات الذكية

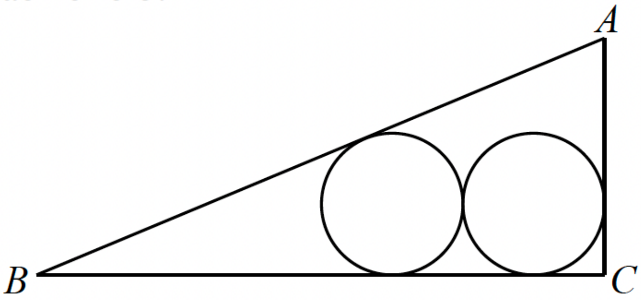
SECTION A

Problem 1 :

Let $P_1P_2...P_n$ be a regular n-gon, where $n \geq 29$. If $\angle P_5P_{27}P_{29} = 165°$, what is the value of n?

Problem 2 :

In the diagram below, ABC is a triangle with $AB = 13$ cm, $BC = 12$ cm and $AC = 5$ cm. Inside this triangle, we consider two equal circles, that are externally tangent to each other, such that one of them is tangent to sides AB and BC, and the other one is tangent to sides BC and AC. What is the length, in cm, of the radius of each circle?

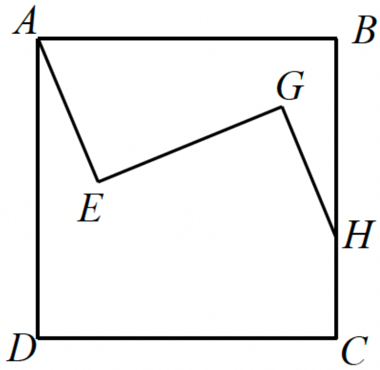


Problem 3 :

What is the sum of all two-digit positive integers $\overline{ab}$, such that $\overline{ab}$ divides the four-digit number $\overline{baba}$?

Problem 4 :

In the diagram below, $ABCD$ is a square. Let E and G be two points inside square $ABCD$ and H be a point on BC such that $\angle AEG = \angle EGH = 90°$ and $EG = BH$. If $AE = 13$ cm and $GH = 12$ cm, what is the side length, in cm, of square $ABCD$?



Problem 5 :

What is the sum of all positive integers n such that $n!$ has exactly 100 trailing zeros?

Problem 6 :

What is the sum of all prime numbers p such that p^2 divides $11^{p^2}+1$?

Problem 7 :

What is the largest real number k such that

$$(x^2+y^2+4x-4y-1)^2+(x^2+y^2-12x-16y-k)^2=0$$

for some ordered pair of real numbers (x, y)?

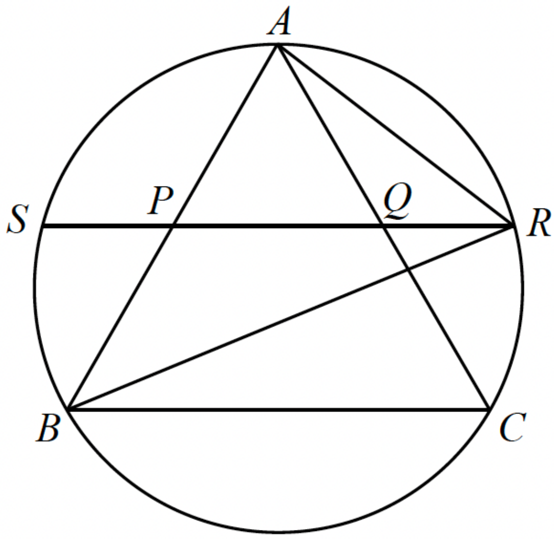
SECTION B

Problem 8 :

Let n be any integer where $n \geq 2$. Now, consider all the fractions in the form $\frac{1}{pq}$, where $\gcd(p, q) = 1$, $0 < p < q \leq n$ and $p + q > n$. Prove that the sum of all such fractions is $\frac{1}{2}$.

Problem 9 :

In the diagram below, circle O is the circumcircle of the equilateral triangle ABC. Points P and Q are the midpoints of AB and AC, respectively. Line PQ intersects circle O at points S and R. If $AR = 10$ cm, what is the length, in cm, of BR?



Problem 10 :

Alex has a fair six-faced die with the fractions $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$ and $\frac{11}{2}$ on its faces. Ben, on the other hand, has a spinning wheel, that is divided into n equal parts. An integer from 0 to $n-1$ is written on each part, with each number appearing exactly once.

Alex rolls his die, and Ben spins his wheel at the same time. Let p be the probability that Ben obtains a greater number than the one Alex obtains. What is the smallest value of n for which p is at least $\frac{1}{2}$?