Assignment 3

Instructions	تعليمات
1) This assignment consists of 6 questions: **Section A:** requires numerical answers only. **Section B:** requires full solutions.	1) يتكون هذا الواجب من 6 أسئلة : **القسم A :** يتطلب إجابات عددية فقط. (4 أسئلة) **القسم B :** يتطلب حلولًا كاملة. (2 أسئلة)
2) Each question in Section A is worth 5 points. No partial credit are given, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found.	2) كل سؤال في **القسم A** يساوي **5 نقاط**. لا تُمنح نقاط جزئية. ويجب ألا تعطي أكثر من عدد الإجابات المطلوب. بالنسبة للأسئلة التي تطلب عدة إجابات، تُمنح الدرجة الكاملة فقط إذا تم العثور على جميع الإجابات الصحيحة.
3) Each question in Section B is worth 20 points. Partial credits may be awarded.	3) كل سؤال في **القسم B** يساوي **20 نقطة**. يمكن منح نقاط جزئية.
4) Diagrams shown may not be drawn to scale.	4) قد لا تكون الرسوم التوضيحية المرفقة مرسومة على مقياس صحيح.
5) You cannot use instruments such as protractors, calculators and electronic devices, smart watches.	5) لا يمكنك استخدام أدوات مثل المنقلة، الآلات الحاسبة، الأجهزة الإلكترونية أو الساعات الذكية

SECTION A

Problem 1 :

Let $P(x)$ and $Q(x)$ be two quadratic polynomials with integer coefficients and their leading coefficients are both 1. It is known that $P(Q(0)) = Q(P(0)) = 1$ and $P(0) + Q(0) = 2$. Find the value of $P(3) + Q(3)$.

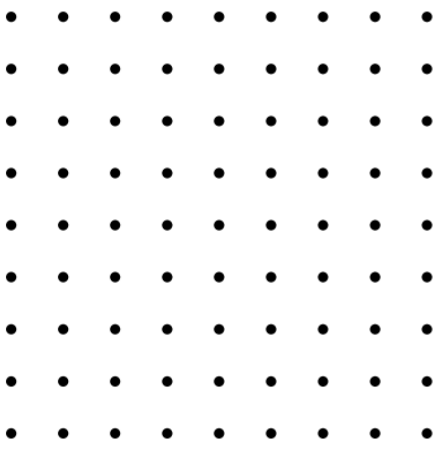
Problem 2 :

Let $a_1, a_2, a_3, \ldots, a_9$ be a random arrangement of the numbers 1, 2, 3, …, 9. What is the greatest possible value of

$$|a_1-\sqrt{3}a_2|+|a_2-\sqrt{3}a_3|+|a_3-\sqrt{3}a_4|+\ldots+|a_8-\sqrt{3}a_9|+|a_9-\sqrt{3}a_1|?$$

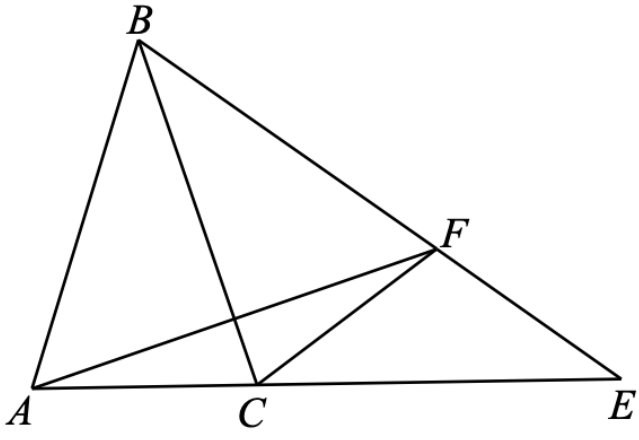
Problem 3 :

There are 81 dots which are arranged in a 9×9 array as shown below. If the distance between any two adjacent dots on the same row or column is 1 cm, determine the number of rectangles that can be formed having an area of 12 cm^2, where all four vertices are among these 81 dots.



Problem 4 :

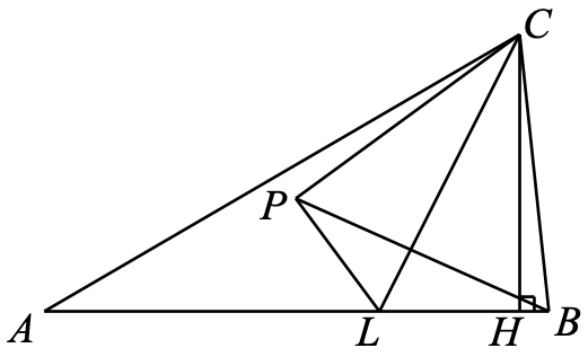
In an isosceles triangle ABC, where $AB=BC$, point E is on the extension of side AC, where C is between A and E and point F is on the segment BE such that $AC=CF=FE$ and $\angle BAF=3\angle FAE$, as shown in the diagram below. Find the measure, in degrees, of $\angle FAE$.



SECTION B

Problem 5 :

Let P be an interior point of acute triangle ABC such that $CP = BP$ and $\angle BPC = 2\angle BAC$. Let the angle bisector of $\angle ACB$ and AB intersect at L and point H be on AB such that $CH \perp AB$, where L is between points A and H, as shown in the diagram below. If $CP = CH = 28$ cm and the area of triangle CPL is 196 cm^2, then find the length, in cm, of LH.



Problem 6 :

Let M and N be non-negative integers such that C_{1010}^{2021} is divisible by $2^M \times 3^N$. Find the sum of all the possible values of $M+N$. (Note: C_{1010}^{2021} refers to the combination of 2021 things taken 1010 at a time without repetition.)