Assignment 6

Instructions	تعليمات
1) This assignment consists of 6 questions: **Section A:** requires numerical answers only. **Section B:** requires full solutions.	1) يتكون هذا الواجب من 6 أسئلة : **القسم A :** يتطلب إجابات عددية فقط. (4 أسئلة) **القسم B :** يتطلب حلولًا كاملة. (2 أسئلة)
2) Each question in Section A is worth 5 points. No partial credit are given, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found.	2) كل سؤال في **القسم A** يساوي **5 نقاط**. لا تُمنح نقاط جزئية. ويجب ألا تعطي أكثر من عدد الإجابات المطلوب. بالنسبة للأسئلة التي تطلب عدة إجابات، تُمنح الدرجة الكاملة فقط إذا تم العثور على جميع الإجابات الصحيحة.
3) Each question in Section B is worth 20 points. Partial credits may be awarded.	3) كل سؤال في **القسم B** يساوي **20 نقطة**. يمكن منح نقاط جزئية.
4) Diagrams shown may not be drawn to scale.	4) قد لا تكون الرسوم التوضيحية المرفقة مرسومة على مقياس صحيح.
5) You cannot use instruments such as protractors, calculators and electronic devices, smart watches.	5) لا يمكنك استخدام أدوات مثل المنقلة، الآلات الحاسبة، الأجهزة الإلكترونية أو الساعات الذكية

SECTION A

Problem 1 :

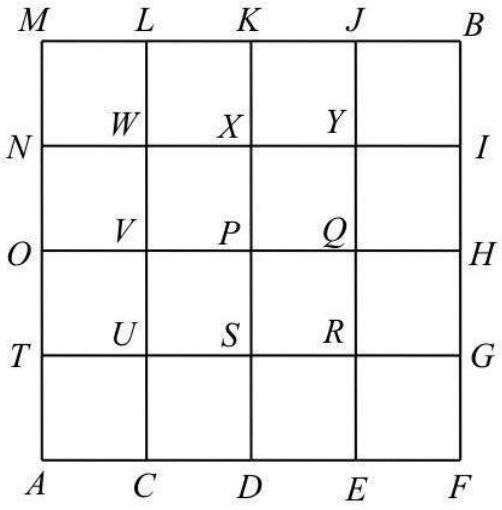
Let x and y be real numbers such that $(2x+\sqrt{1+4x^2})(3y+\sqrt{1+9y^2})=1$.
Find the numerical value of $(2x+3y)^2$.

Problem 2 :

Let p and q be prime numbers such that $p^2 + 3pq + q^2$ is the square of an integer.
Find the largest possible value of $p + q$.

Problem 3 :

The diagram given below shows an underground maze in the form of a 4×4 square. A snake, initially at position P, starts crawling along the path P-Q-R-S in a loop. A mouse, initially at position A, has to reach position B, moving across the maze. The mouse can only move vertically up or horizontally right. If the snake and the mouse reach the same position at the same time, the snake will eat the mouse; also if the two cross each other along a path, the snake will eat the mouse (the snake and the mouse move at the same speed). Given that the mouse and the snake start moving at the same time, what is the number of safe paths for the mouse to move from A to B?
(For example, A-C-D-E-R-G-H-I-B is a safe path for the mouse, since when the mouse reaches R, the crawling snake will be at point P)



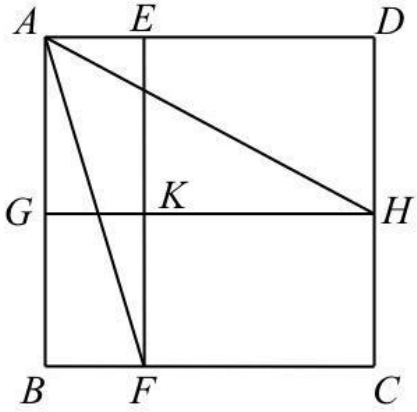
Problem 4 :

Let $P(x) = x^4 + ax^3 + bx^2 + cx + d$, where a, b, c and d are real constants.
Suppose $P(1) = 7$, $P(2) = 52$ and $P(3) = 97$. Find the value of $\dfrac{P(9) + P(-5)}{4}$.

SECTION B

Problem 5 :

Let $ABCD$ be a square. E and F are points on AD and BC respectively such that $EF//AB$. G and H are points on AB and DC respectively such that $GH//AD$. EF and GH intersect at K. If the area of $KFCH$ is equal to twice that of $AGKE$, find the measure, in degrees, of $\angle FAH$.



Problem 6 :

. Let $f(x)$ and $g(x)$ be distinct quadratic polynomials such that the leading coefficients of both polynomials are equal to 1 and

$$f(1)+f(2017)+f(2017^2)=g(1)+g(2017)+g(2017^2).$$

Find x if $f(x)=g(x)$.